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Cooperative game theory – a scope for electricity supply security?

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Abstract. The objective of this paper is to introduce a method, by means of cooperative game theory, for distributing a potential gain ascribing to voluntary pooled firm specific random resources. By modelling a supply game akin to linear stochastic production games, a core allocation yield economic signals which may promote incentives to perform investments that will diversify the market's supply technology mix and stimulate efficient demand-response. Liberalized electricity markets with inherent environmental risk stand out as areas of applicability.

Key words: Cooperative Games; Stochastic Programming; Core; Endogenous Risk; Electricity Supply Security
JEL classification: C71, C73; D81; Q48

1. Introduction

Several so-called deregulated electricity markets have experienced energy or capacity shortages, and some have also had to deal with blackouts. Well-known electricity crises include Brazil (2001), California (2000/2001) and Chile (1998/1999), see e.g. Lock (2005), Joskow (2001) and Watts and Ariztía (2002) for overviews and comparisons. These and similar incidents in other countries have brought the issue of supply security in deregulated electricity markets to the foreground of political debate, where deregulation, per se, often is blamed for the lack of adequate level of investments in power production capacity (and/or infrastructure) – which is taken to be the underlying factor inducing these crises.

In theory, deregulated electricity markets should provide appropriate investment incentives to market participants. Conditions that are needed to fulfill this theoretical prediction include unregulated prices to consumers, so that the imbalance between supply and demand is revealed, and free entry of generating plants, in order to bring forth the most efficient project. However, in practice high and volatile electricity prices are often met with populist measures such as price-ceilings, and the realization of investments in supply capacity must overcome political risk, environmental restrictions and challenges caused by the lumpiness of capacity additions, c.f. Joskow

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(2005) for a thorough discussion of supply security problems in deregulated electricity sectors induced by flaws in the corresponding liberalization process. As a result, the interim period in which electricity supply butt against the capacity limit before new generating stations are in operation, is likely to be prolonged. This situation is particularly demanding for markets depending largely on hydro electric power, as a tighter energy balance may coincide with periods of unexpected precipitations shortfall.

Since the average annual electricity price levels are expected to fluctuate in accordance with occasional wet and dry years, a sharp increase in prices can be triggered by an incidental dry year situation or a tighter sypply-demand balance, or a mix of both. Therefore, it may be difficult for policymakers to detect the true cause of an upcoming electricity supply crisis and, even more difficult, to explain to voters the beneficent effect of high prices in providing incentives for capacity investments. Moreover, as these markets steadily move towards their short-term production capacity limit, inflow may just as well peak, possibly bringing prices down below the normal level and wiping any recent emergency situation – and the reasons for it – temporarily off voters' memory. Additionally, the positive probability of inflow peaks exerts a downward pressure on forward prices, thus dampening potential investments in new generating plants or reserve capacity. Then, as demand grows, unexpected precipitation shortfalls of lesser magnitude may bring about price spikes, accompanied by new warnings of supply emergency and government pleas for electricity demand moderations. As a result, market deregulation will be hotly debated and the pressure on authorities to re-regulate the market may gain strength, see e.g. Hogan (2002) for a discussion on re-regulation processes.

Markets as described above comprise both regulatory risk and hydro inflow risk. Therefore, policy instruments that contribute to smoothening the bumps on the transition path from short-run supply deficit to long-run resource adequacy – without distorting the price mechanism – will be of high economic value. The objective of this paper is to utilize and further develop two-stage stochastic linear production games, c.f. Owen (1975), Sandsmark (1999) and Flåm (2002), to the benefit of electricity (energy) supply security. More specifically, to avail supply security I shall furnish a stochastic cooperative supply game that may yield incentives for market participants to diversify the aggregate supply technology mix to reduce inflow risk, and produce efficient demand-response to provide supply insurance. To achieve such beneficial results one must establish a grand coalition of electricity producers and large consumers by pooling individual random resources (supply and demand), and divide the aggregate minimum cost associated with first-stage capacity settlements and subsequent dispatch in accordance with a core solution derived by means of optimal dual variables. The salient feature of the suggested core allocation is that coalition members need not bring negatively correlated resources to the joint enterprise to be better off compared to self-sufficiency. Moreover, firms with scarce resources are amply rewarded.

Cooperative game theory has previously been applied to problems related to power investments, c.f. Gately (1974), and transmission planning c.f. Contreras and Wu

(1999, 2000) and Zolezzi and Rudnick (2002), but without having considered stochastic events. Stochastic cooperative games are otherwise analyzed in Suijs and Borm (1999).

The rest of the paper is organized as follows. Section 2 presents the two-stage stochastic supply game and defines an efficient allocation of associated expected costs. The paper's main contribution is the characterization of the optimal solution in terms of endogenous risk, which is displayed in Section 3, along with a discussion of possible implications for electricity (energy) supply security. Numerical illustrations are presented in Section 4 and Section 5 concludes.

2. The two-stage stochastic supply game

Let there be a fixed, finite set I of electricity suppliers with generating plants located at one or more origins $o \in O$. (It is common for electricity companies to own power generating units at several nodes of a transmission network.) A firm does not, however, need to have a plant at each origin o. All firms supply electricity to customers distributed at various destinations $d \in D$. By assumption the sets O and D are finite and disjoint.

Electricity producers frequently face uncertainty of various sorts. Therefore, it is likely that some decisions must be made before important information is known. The problem facing the producers here is to determine the generating capacity so as to meet the customers' electricity demand at minimum supply costs before they know the exact demand and available input, i.e., hydro inflow or wind power. The activities of the firms thus take place over two stages: first, supply commitments (capacity decisions) are made, second, production and dispatch take place. Consequently, the first-stage decision $x^1 \in \mathbb{R}^{n_1}_+$ denotes an "irreversible" choice made *here-and-now* under uncertainty, and the second-stage decision $x^2(\xi) \in \mathbb{R}^{n_2}_+$ denotes the adjustment undertaken when the random outcome $\xi \in \Xi$ becomes known, where Ξ denotes a finite probability space of elementary outcomes ξ, each one happening with prescribed probability $p(\xi)$. The probabilities are assumed to be commonly known.

Let firm i have immediate supply $\mathcal{S}_i^{1o}$ already available at origin $o \in O$, and future contingent supply $\mathcal{S}_i^{2o}(\xi)$ at the same site in case $\xi \in \Xi$ should happen. In addition, the firm faces demand $\mathcal{D}_i^d(\xi)$ at destination $d \in D$. Then, let

$$\begin{aligned} x^{od} &:= \text{contracted supply (capacity) from } o \to d \text{ before } \xi \text{ is known,}\\ x^{od}(\xi)^+ &:= \text{reserve power } o \to d \text{ in case } \xi \text{ causes excess demand, and}\\ x^{od}(\xi)^- &:= \text{superfluous power } o \leftarrow d \text{ in case } \xi \text{ causes excess supply.}\end{aligned}$$

The constraints of firm i thus assumes the form

$$\left.\begin{array}{llllll} \sum_{d\in D} x^{od} & & & \leq & \mathcal{S}_i^{1o}, & \forall o,\\ \sum_{d\in D}(x^{od} & + \; x^{od}(\xi)^+ & - \; x^{od}(\xi)^-) & \leq & \mathcal{S}_i^{2o}(\xi), & \forall o,\xi,\\ \sum_{o\in O}(x^{od} & + \; x^{od}(\xi)^+ & - \; x^{od}(\xi)^-) & \leq & \mathcal{D}_i^{d}(\xi), & \forall d,\xi,\\ x^{od}, & x^{od}(\xi)^+, & x^{od}(\xi)^- & \geq & 0, & \forall o,d,\xi, \end{array}\right\} \tag{1}$$

Thus the problem of firm i amounts to

$$\text{minimize} \left\{ c^1 \cdot x^1 + \sum_{\xi \in \Xi} p(\xi) c^2(\xi) \cdot x^2(\xi) \right\} \text{subject to (1)} \tag{2}$$

where the shorthand notation $x^1 := (x^{od})_{o \in O, d \in D}$, $x^2(\xi) := (x^{od}(\xi)^+, x^{od}(\xi)^-)_{o \in O, d \in D}$, $c^1 := (c^{od})_{o \in O, d \in D}$, and $c^2(\xi) := (c^{od}(\xi)^+, c^{od}(\xi)^-)_{o \in O, d \in D}$ is used. By assumption $c^{od} < c^{od}(\xi)^- < c^{od}(\xi)^+$, which reflects the increased cost of short-term recourse actions, and implies that reacting to a supply deficit is more costly than handling excess supply.

However, since electricity is (by most standards) a homogeneous good, customers are indifferent to the origins of supply. Therefore, taking advantage of joint possibilities, some firms can meet the contracts on behalf of other firms. More specifically, a coalition $S \subseteq I$ of firms can pool their vectors of first and second stage individual resources

$$\mathcal{S}_S^{1o} := \sum_{i \in S} \mathcal{S}_i^{1o}, \; \mathcal{S}_S^{2o}(\xi) = \mathcal{S}_S^{1o} + \sum_{i \in S} \mathcal{S}_i^{2o}(\xi) \text{ and } \mathcal{D}_S^d(\xi) := \sum_{i \in S} \mathcal{D}_i^d(\xi),$$

and solve

$$\left.\begin{array}{lllll} \text{minimize} & c^1 \cdot x^1 + \sum_{\xi \in \Xi} p(\xi) c^2(\xi) \cdot x^2(\xi) & & & \\ \text{subject to} & \sum_{d \in D} x^{od} & \leq & \mathcal{S}_S^{1o}, & \forall o, \\ & \sum_{d \in D} (x^{od} + x^{od}(\xi)^+ - x^{od}(\xi)^-) & \leq & \mathcal{S}_S^{2o}(\xi), & \forall o, \xi, \\ & \sum_{o \in O} (x^{od} + x^{od}(\xi)^+ - x^{od}(\xi)^-) & \geq & \mathcal{D}_S^d, & \forall d, \xi, \\ \text{and} & x^{od}, x^{od}(\xi)^+, x^{od}(\xi)^- & \geq & 0, & \forall o, d, \xi, \end{array}\right\} \tag{3}$$

Let $c(S)$ denote the associated minimal cost of (3). Our concern is whether the total cost $c(I)$ can be achieved and divided fairly. Taking the advice of cooperative game theory an allocation $u = (u_i)_{i \in I}$ of total cost $c(I)$ should lie in the *core*. Then consider the *cooperative transferable utility game* (I, c) with characteristic function $S \mapsto c(S)$, see e.g. Shubik (1982) for a text-book reference to cooperative game theory.

Definition 1. *(Core) A cost allocation $u = (u_i)_{i \in I}$ is an element in the core of the cooperative game (I, c) if*

$$\sum_{i \in S} u_i \leq c(S), \; \textit{for all } S \subset I, \quad \textit{and} \quad \sum_{i \in I} u_i = c(I).$$

The inequalities imply coalitional stability: no individual ($S = (\{i\})$) or group of players can do better by themselves. The equation accounts for Pareto efficiency (group rationality). Subadditivity is necessary for non-emptyness of the core:

$$c(S) + c(S') \geq c(S + S') \text{ for disjoint coalitions } S, S' \subset I.$$

This condition evidently holds in our case. Moreover, the cost sharing game is balanced, which suffices for non-emptiness of the core, c.f. Bondareva (1962) and Shapley (1967). From the analysis of deterministic production games in Owen (1975) and production games under uncertainty in Sandsmark (1999) we get forthwith

Theorem 1. *(Non-empty core of stochastic supply efficiency game) Assume that the linear program (3) is feasible and has a finite optimal value for $S = I$. Then (3) defines a cooperative stochastic supply game which is totally balanced.* □

Following Owen (1975) core elements are found in terms of solutions associated with the dual of program (3) when $S = I$. To apply this method here, we need to require that the second-stage problem is feasible for any first-stage decision x^1 and all $\xi \in \Xi$. This is, however, not at trivial assumption, c.f. Kall and Wallace (1994).

Proposition 1. *Without any loss, multiply lines three and four of (3) by $p(\xi) > 0$. Then let the vectors λ^1 and $\lambda^2 := (\lambda^2(\xi))_{\xi\in\Xi}$ solve the dual problem associated with (3) when $S = I$, and distribute total cost such that each player $i \in I$ receives*

$$u_i := \mathcal{S}_i^1 \cdot \lambda^1 + \sum_{\xi\in\Xi} p(\xi) \left[\mathcal{SD}_i^2(\xi) \cdot \lambda^2(\xi)\right], \tag{4}$$

where $\mathcal{S}_i^1 := (\mathcal{S}_i^{1o})_{o\in O}$, $\mathcal{SD}_i^2 := (\mathcal{S}_i^{2o}(\xi), \mathcal{D}_i^{2d}(\xi))_{o\in O, d\in D}$, $\lambda^1 := (\lambda^{1o})_{o\in O}$, and $\lambda^2(\xi) := (\lambda^{2o}(\xi), \lambda^{2d}(\xi))_{o\in O, d\in D}$. Then the imputation $u = (u_i)_{i\in I}$ is an element in the core of the stochastic cost-sharing game defined in (3).

Proof: The minimum of the dual stochastic supply problem will equal the optimal value $c(S)$, and by letting $\lambda^1, \lambda^2 := (\lambda^2(\xi))_{\xi\in\Xi}$ be the optimal solution vectors for the dual program when $S = I$, we get

$$c(I) = \mathcal{S}_I^1 \cdot \lambda^1 + \sum_{\xi\in\Xi} p(\xi) \left[\mathcal{SD}_I^2(\xi) \cdot \lambda^2(\xi)\right] \text{ and} \tag{5}$$

$$c(S) \geq \mathcal{S}_S^1 \cdot \lambda^1 + \sum_{\xi\in\Xi} p(\xi) \left[\mathcal{SD}_S^2(\xi) \cdot \lambda^2(\xi)\right], \text{ for any } S. \tag{6}$$

Consequently, distributing total costs $c(I)$ by the rule

$$u_i := \mathcal{S}_i^1 \cdot \lambda^1 + \sum_{\xi\in\Xi} p(\xi) \left[\mathcal{SD}_i^2(\xi) \cdot \lambda^2(\xi)\right], \text{ for all } i \in I,$$

we have, for any S,

$$\sum_{i\in S} u_i = \sum_{i\in S} \left(\mathcal{S}_i^1 \cdot \lambda^1 + \sum_{\xi\in\Xi} p(\xi) \left[\mathcal{SD}_i^2(\xi) \cdot \lambda^2(\xi)\right]\right)$$

yielding by (5) and (6), respectively,

$$\sum_{i \in I} u_i = c(I) \text{ and } \sum_{i \in S} u_i \leq c(S)$$

Thus the resulting allocation $u = (u_i)_{i \in I}$ belongs to the core of (3). □

When the grand coalition's costs are shared as suggested above, each firm is charged according to individual expected resources (electricity supply and demand), evaluated by the corresponding optimal dual variables.

3. Endogenous risk

Compared to the case of self-sufficiency, the pooling of individual resources increases the flexibility of electricity supply under uncertainty, due to more efficient use of resources. Moreover, cooperation reduces risk without necessarily increasing expected costs.

Proposition 2. *A set of agents $i \in I$ with individual objective (2) and stochastically independent second stage resource vectors $\mathcal{SD}_i^2(\xi)$, $\xi \in \Xi$, i.e. $cov\left(\mathcal{SD}_i^2,\ \mathcal{SD}_j^2\right) = 0$ $\forall i, j \in I$, can reduce their risk by pooling individual resources, then solve (3) for the grand coalition and distribute the joint cost as defined by (4).*

Proof: By Proposition 1, the optimal cost sharing rule is defined by

$$u_i := \mathcal{S}_i^1 \cdot \lambda^1 + E\left[\mathcal{SD}_i^2 \cdot \lambda^2\right].$$

Further, the second-stage expected enumeration can be written

$$E\left[\mathcal{SD}_i^2 \cdot \lambda^2\right] = E\left[\mathcal{SD}_i^2\right] \cdot E\left[\lambda^2\right] + \sum_{k \in K} cov\left(\mathcal{SD}_i^{2k},\ \lambda^{2k}\right) \tag{7}$$

where K denote the index set of second stage supply-demand vectors. Since, by duality $cov\left(\mathcal{SD}_i^{2k},\ \lambda^{2k}\right) \neq 0$, then most likely

$$\sum_{k \in K} cov\left(\mathcal{SD}_i^{2k},\ \lambda^{2k}\right) \neq 0. \tag{8}$$

The difference between $E\left[\mathcal{SD}_i^2 \cdot \lambda^2\right]$ and $E\left[\mathcal{SD}_i^2\right] \cdot E\left[\lambda^2\right]$ defines the individual risk premium. □

Now, include in the grand coalition I a player j with second stage resource vector $\mathcal{SD}_j^2(\xi)$, $\xi \in \Xi$, with stochastically independent outcomes, that is $cov\left(\mathcal{SD}_j^2,\ \mathcal{SD}_i^2\right) = 0$ $\forall i \in I, j$. Then probability mass is moved from the players' least preferred outcome to more agreeable outcomes, thus reducing the variance of the stochastic cooperative game defined in (3) for the grand coalition $I \ni j$ compared to solving (3) for the grand coalition where $j \notin I$.

Corollary 1. *(Self-protection) Adding players* $j \in J$, $J \cap I = \varnothing$,*with stochastically independent resource vectors* $\mathcal{SD}_j^2(\xi)$, $\xi \in \Xi$, *such that* $cov\left(\mathcal{SD}_j^2,\ \mathcal{SD}_i^2\right) = 0\ \forall i \in I, j \in J$, *will reduce risk when solving (3), ceteris paribus.* □

If, on the other hand the grand coalition includes a player with an individual resource vector that are negatively correlated, i.e. $cov\left(\mathcal{SD}_i^2,\ \mathcal{SD}_j^2\right) \neq 0$ for any $i, j \in I$, the least preferred random outcomes become more agreeable, should they be realized.

Corollary 2. *(Self-insurance) If players* $i \in I$ *have negatively correlated individual second stage resource vectors* $\mathcal{SD}_i^2(\xi)$, $\xi \in \Xi$, *such that* $cov\left(\mathcal{SD}_i^2,\ \mathcal{SD}_j^2\right) \neq 0$ $i, j \in I$, *then the benefit of establishing the grand coalition - in terms of insurance - is obvious.* □

Note here the relationship of Corollary 1 and Corollary 2 with the literature on endogenous risk and moral hazard. Following Ehrich and Becker (1972) actions that reduce specific risks, in terms of increasing the probability of favorable outcomes, are labelled self-protection and actions that reduce the severity of specific risks, are labelled self-insurance.

The applicability of the above results are illustrated in the numerical examples in the subsequent section. First, I discuss some implications of the above results on electricity supply security.

3.1. Implications for supply security.

Diversification. If the authorities, or the industry itself, arrange a cooperative game of producers who pool random resources to minimize and distribute joint expected cost as suggested by (3) and (4), respectively, it is reasonable to assume that repeated play would promote investments in supply technologies that create diversification benefits in terms of supply security, or alternatively, resource adequacy. In electricity markets relying heavily on supply technologies with inherent environmental risk, such as e.g. the hydro dominated Nordic, Brazilian or New Zealand electricity markets, one would expect that capacity expansions that include wind power, thermal power or bio-energy will have some propitious effects. Today, deregulated electricity markets do not provide efficient methods that award firms investing in technologies which contribute to the diversification of the market's portfolio of generating facilities.

A related argument is conveyed in Joskow and Tirole (2005), which is applied to merchant transmission investments under uncertainty. However, here I also expose a benefit stemming from reduced risk, a kind of moral hazard or self-protection action, that is realized independently of any diversification potential. That is, in case the pooled random resources are stochastically independent, there is still a possibility to extract a joint expected benefit, made visible by (8), due to reduced variance. Short-term risk reduction benefits may, therefore, yield long term diversification benefits.

Demand-response. Efficient demand-response is stressed as one of the key-factors contributing to increased supply security in deregulated markets in the longer term. Analyses of voluntary demand reductions accompanying electricity supply shortages – either due to government campaigns, higher electricity prices or both – conclude that such customer responses have played a significant role in alleviating the severeness of supply crises, see e.g. Goldman et al. (2002). They analyze the California electricity crisis in 2000-2001 and, controlling for changes in weather and economic conditions, conclude that the amount of customer load reductions stemming from deliberate demand responses to the evolving electricity shortage, contributed to a significant reduction in the number of rolling blackouts predicted beforehand. Also during the supply shock that hit the Nordic electricity market in 2002/2003, a considerable reduction in demand from Norwegian customers, contributed to preventing rationing as the authorities warned would be necessary, see von der Fehr et al. (2005) and ECON (2003). For this to take effect, end-user prices must be allowed to reflect scarcity of supply. Moreover, if market participants know that price spikes originating from supply deficits will not be inhibited when short run reliability criteria are effectuated, incentives for investing in new generating facilities are not hampered. This is important because investors must depend on relatively high prices in some hours to be able to cover both the operating costs and the investment costs of new capacity, c.f. Joskow (2007).

But, the energy intensive industry, which typically consists of firms that compete in the world market, is one of the most influential lobbyists for price (re-)regulations. If electricity prices in deregulated electricity markets on average tend to be higher than in markets with regulated electricity prices, one would expect that these consumers eventually move to production sites with lower input prices. At least, this is their threat. However, as the above discussion reveal, demand-response may be important in situations of supply deficits. So, obviously, there is a dilemma between allowing market prices to peak and to keep the potentially useful large consumers.

A question then emerges: How can large flexible customers be efficiently compensated for providing a positive externality? If we establish a cooperative game, include in the grand coalition the firms constituting the power intensive industry and implement the solution prescribed by (3) and (4), large consumers that reduce their demand when supply is scarce, will be effectively compensated. This may be an efficient addition to markets for reserve capacity or a more economically viable alternative to re-regulation or subsidizing demand reductions in advance, which is what Ruff (2002) warns may be paying twice for the same thing. More specifically, when joining the grand coalition these firms are not paid to reduce consumption when resources are low, which they might do anyway for free, but they are benefited through the sharing of an insurance benefit that emerges because of potentially correlated random outcomes, without disturbing the market mechanism.

4. NUMERICAL ILLUSTRATIONS

Let there be three electricity producing firms, which supply customers at three destinations from one generation plant each; i.e., $i = o$ and $I = O = D = \{1, 2, 3\}$. Endow each firm with first-stage electricity supply $\mathcal{S}_1^1 = (250, 0, 0)$, $\mathcal{S}_2^1 = (0, 500, 0)$, and $\mathcal{S}_3^1 = (0, 0, 410)$.

4.1. Example of self-protection. Suppose that second-stage supply $\mathcal{S}_i^{2o}(\xi)$ and demand $\mathcal{D}_i^{2d}(\xi)$ are independent for any firm i (and also across firms) with uniform, two-point distributions as displayed in Table 1 and Table 2.

Table 1:

Contingent supply at various origins

$o \backslash i$	$\mathcal{S}_1^2(\cdot)$		$\mathcal{S}_2^2(\cdot)$		$\mathcal{S}_3^2(\cdot)$	
	high	low	high	low	high	low
1	300	250	0	0	0	0
2	0	0	600	500	0	0
3	0	0	0	0	500	410

Table 2:

Contingent demand at various destinations

$d \backslash i$	$\mathcal{D}_1^2(\cdot)$		$\mathcal{D}_2^2(\cdot)$		$\mathcal{D}_3^2(\cdot)$	
	high	low	high	low	high	low
1	130	110	300	250	0	0
2	120	90	0	0	210	150
3	0	0	200	150	200	150

As a consequence of the supply and demand pattern displayed above, this example has $2^9 = 512$ equally possible outcomes ξ. That is, we get an event space $\Xi = \{high, low\}^9$ with uniform probability distribution $p(\xi) = \frac{1}{512}$ for all ξ.

To reveal only the gains of the cooperative enterprise that is due to risk reduction, I fix the values of the cost matrices, so that all contain the same value, here set to $c^{od} = 10$, $c^{od-} = 12$ and $c^{od+} = 30$ for all links o, d. Neither the underlying network matrix nor the cost parameters are stochastic.

Solving the minimum cost dispatch model defined (3) for the specifications given above[1], the grand coalition incurs minimum expected cost $c^A(I) = 11350$, which includes a certain amount of 10600, corresponding to the optimal first-stage shipment $\bar{x}^1$:

[1] The numerical examples are modelled and solved using the software tool GAMS.

$o\backslash i$	1	2	3
1	0	0	250
2	410	0	90
3	0	300	10

The superscript A is applied to distinguish these numerical results from the results in the subsequent example, which are denoted by superscript B.

When splitting the joint cost $c^A(I)$ as defined by (4), a core allocation becomes

$$u^A = (2375.000,\ 4938.477,\ 4036.523), \tag{9}$$

with associated individual risk premiums, $r_i = \sum_{k \in K} cov^A\left(\mathcal{SD}_i^{2k}, \lambda^{2k}\right)$ extracted from (7), for this example:

$$r_1^A = 125.000\ ,\ r_2^A = 438.477 \text{ and } r_3^A = 486.523.$$

Here the set K has nine elements corresponding to the supply and demand pattern depicted in the tables above.

In contrast to splitting the joint cost $c^A(I)$, if the firms choose not to cooperate, the individual expected costs are $c(1) = 2500$, $c(2) = 5000$, and $c(3) = 4100$, which implies that

$$u_i^A < c(i) \text{ for all } i \in I.$$

This numerical example displays a positive risk premium for all players in the grand coalition despite that the individual random resources are stochastically independent. Joining the grand coalition reduces risk and may be labelled an action of self-protection.

4.2. Example of self-insurance. Now suppose that a large consumer is invited to join the supply game, i.e., $i \in I = \{1, 2, 3, 4\}$, $I = O$, but still $D = \{1, 2, 3\}$. This large consumer, player 4, may be an energy-intensive firm, e.g., a producer of aluminum. Due to stochastic market fluctuations, such as the market price of aluminum, bauxite or other, this firm may reduce its peak load demand by 50. If we assume that this amounts to the difference between producer 2's high and low contingent demand at destination 1, we can adjust producer 2's demand at destination 2 to be constant at 250. Also, depending on the outcome of ξ, when the energy intensive firm reduces its demand by 50 this amount becomes available to other consumers, which I choose to model as a contingent supply to player 4 at origin 4, see the adjusted tables of contingent supply and demand, Table 3 and Table 4.

Table 3:
Contingent supply at various origins

$o\backslash i$	$\mathcal{S}_1^2(\cdot)$		$\mathcal{S}_2^2(\cdot)$		$\mathcal{S}_3^2(\cdot)$		$\mathcal{S}_4^2(\cdot)$	
	high	low	high	low	high	low	high	low
1	300	250	0	0	0	0	0	0
2	0	0	600	500	0	0	0	0
3	0	0	0	0	500	410	0	0
4	0	0	0	0	0	0	50	0

Table 4:
Contingent demand at various destinations

$d\backslash i$	$\mathcal{D}_1^2(\cdot)$		$\mathcal{D}_2^2(\cdot)$		$\mathcal{D}_3^2(\cdot)$	
	high	low	high	low	high	low
1	130	110	250	250	0	0
2	120	90	0	0	210	150
3	0	0	200	150	200	150

Note that this modified example also has $2^9 = 512$ equally possible outcomes ξ, and that the first and second stage unit costs are the same as above. The realization of Individual resources are, however, no longer uncorrelated.

Solving again the minimum cost dispatch model defined in (3) for the adjusted specifications given here, the grand coalition incurs minimum expected cost $c^B(I) = 10676.25$, which includes a certain amount of 10300, corresponding to the optimal first-stage shipment $\bar{x}^1$:

$o\backslash d$	1	2	3
1	0	240	0
2	380	60	0
3	0	0	350

Splitting the joint cost $c^B(I)$ as defined by (4), the adjusted core allocation becomes

$$u^B = (2353.750,\ 4404.102,\ 3989.883, -71.484).$$

Comparing these values with the core allocation for the grand coalition without the large consumer (9), we see that

$$u_i^B < u_i^A \text{ for all } i = 1, 2, 3,$$

implying that all producers benefit from having included the large consumer in the grand coalition, even though the large consumer is compensated for its participation by an amount 71.484. The individual risk premiums r_i^B for this example are

$$r_1^B = 103.750,\ r_2^B = 154.102,\ r_3^B = 439.883 \text{ and } r_3^B = -573.633,$$

which reveal a negative risk premium for the large consumer. Consequently, this player require a compensation to participate in the grand coalition – a demand which is complied with via the core solution.

Due to the random resources of the large consumer not being stochastically independent of the random resources of the producers, establishing the grand coalition in this example provides more agreeable outcomes and may be labelled actions of self-insurance.

5. Conclusion

Having access to powerful optimizing tools, cooperative games under uncertainty have become more accessible for solving real world problems. This paper aims at displaying some beneficial characteristics of cooperative solutions for deregulated electricity markets inclined to experiencing supply deficits.

Electricity markets are, however, very complex and difficult to model properly. Admittedly, the cooperative supply game provided here is naive in this respect. Rather than including more realistic data, I think that providing more realistic assumptions, such as an oligopolistic market environment, would be of more practical use. Such an endeavour would partly rely on an extension of the regional oligopoly in Flåm and Jourani (2003) to encompass random resources. This is left for a subsequent paper.

Other interesting, but complicated improvements would be to model asymmetric information or to analyze the consequences of having horizontally or vertically integrated firms, which are common features of deregulated electricity markets.

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